Christ Church Grammar School

2020
TEST 2

MATHEMATICS SPECIALIST Year 12

Section Two:
Calculator-assumed

Your name SOLUTIONS

Teacher's name _____

Time and marks available for this section

Reading time for this section:	2 minutes
Working time for this section:	20 minutes
Marks available:	20 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet using a blue/black pen. Do not use erasable or gel pens

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 4 **(4 marks)**

A line, L, contains the points $(-1, 1, -4)$ and $(5, 4, -1)$. A plane, P, is defined by the equation $x + 2y - z = 11$. P, intersects with the line, L, at the point $(3, 3, -2)$.

Determine the vector equation of the plane containing L which is perpendicular to P.

direction of line $\begin{pmatrix}6\\3\\3\end{pmatrix}$ or $\begin{pmatrix}2\\1\\1\end{pmatrix}$ $\therefore \underset{\sim}{r}_L = \begin{pmatrix}-1\\1\\-4\end{pmatrix} + \lambda\begin{pmatrix}6\\3\\3\end{pmatrix}$

or $\underset{\sim}{r}_L = \begin{pmatrix}-1\\1\\-4\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\1\end{pmatrix}$ ✓ determines equation of line

✓ determines direction of Line

$\underset{\sim}{n} = \begin{pmatrix}1\\2\\-1\end{pmatrix} \times \begin{pmatrix}6\\3\\3\end{pmatrix}$ or $\underset{\sim}{n} = \begin{pmatrix}1\\2\\-1\end{pmatrix} \times \begin{pmatrix}2\\1\\1\end{pmatrix}$

$= \begin{pmatrix}9\\-9\\-9\end{pmatrix}$ ✓ $= \begin{pmatrix}3\\-3\\-3\end{pmatrix}$

calculates cross product correctly

$\underset{\sim}{r}.\underset{\sim}{n} = \underset{\sim}{a}.\underset{\sim}{n}$

$\underset{\sim}{r}.\begin{pmatrix}9\\-9\\-9\end{pmatrix} = \begin{pmatrix}3\\3\\-2\end{pmatrix}.\begin{pmatrix}9\\-9\\-9\end{pmatrix}$ or $\underset{\sim}{r}.\begin{pmatrix}3\\-3\\-3\end{pmatrix} = \begin{pmatrix}3\\3\\-2\end{pmatrix}.\begin{pmatrix}3\\-3\\-3\end{pmatrix}$ or $\underset{\sim}{r}.\begin{pmatrix}1\\-1\\-1\end{pmatrix} = 2$

$\underset{\sim}{r}.\begin{pmatrix}9\\-9\\-9\end{pmatrix} = 18$ $\underset{\sim}{r}.\begin{pmatrix}3\\-3\\-3\end{pmatrix} = 6$

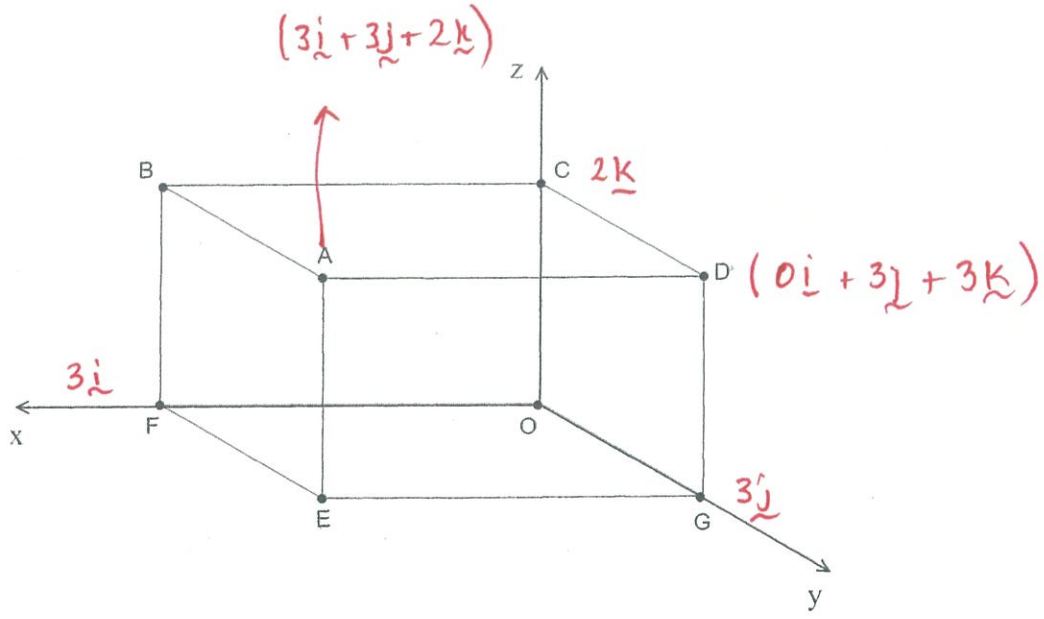
✓ determines vector equation of the plane.

accept $\underset{\sim}{r} = \begin{pmatrix}3\\3\\-2\end{pmatrix} + \lambda\begin{pmatrix}6\\3\\3\end{pmatrix} + \mu\begin{pmatrix}1\\2\\-1\end{pmatrix}$

See next page

Question 5 **(8 marks)**

A right rectangular prism, with square base OGEF, is shown below.
Point O is the origin and points F, G, C have respective position vectors $3\underset{\sim}{i}, 3\underset{\sim}{j}$ and $2\underset{\sim}{k}$.



(a) Determine the vector equation of the line that passes through F and D. (2 marks)

$\overrightarrow{FD} = \begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix}$ ✓ correctly calculates direction

states vector equation of a line ✓ $\therefore \quad \underset{\sim}{r} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix}$ or $\underset{\sim}{r} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix}$.

(b) Determine the Cartesian equation of the plane perpendicular to FD that passes through A. (3 marks)

$\overrightarrow{FD}$ is normal to plane. ✓ uses $\overrightarrow{FD}$ as ⊥ to plane

$$\underset{\sim}{r} \cdot \begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix} = \underset{\sim}{a} \cdot \underset{\sim}{n}$$

$$\underset{\sim}{r} \cdot \begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix}$$

$$\underset{\sim}{r} \cdot \begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix} = 4 \text{ ✓} \quad \therefore \quad -3x + 3y + 2z = 4 \text{ ✓}$$

correctly calculates dot product

states Cartesian equation.

See next page

Question 5 continued

(c) Determine the point of intersection between the line in (a) and the plane in (b). (3 marks)

(a) $\underset{\sim}{r} = \begin{pmatrix} 3-3\lambda \\ 3\lambda \\ 2\lambda \end{pmatrix}$ intersect with (b) $\underset{\sim}{r} \cdot \begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix} = 4$.

$\therefore \begin{pmatrix} 3-3\lambda \\ 3\lambda \\ 2\lambda \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix} = 4$ ✓ subst (a) into (b).

Solving $\lambda = \frac{13}{22}$ ✓ solves for λ

if $\lambda = \frac{13}{22}$ then $\underset{\sim}{r} = \begin{pmatrix} 3-3\left(\frac{13}{22}\right) \\ 3\left(\frac{13}{22}\right) \\ 2\left(\frac{13}{22}\right) \end{pmatrix}$

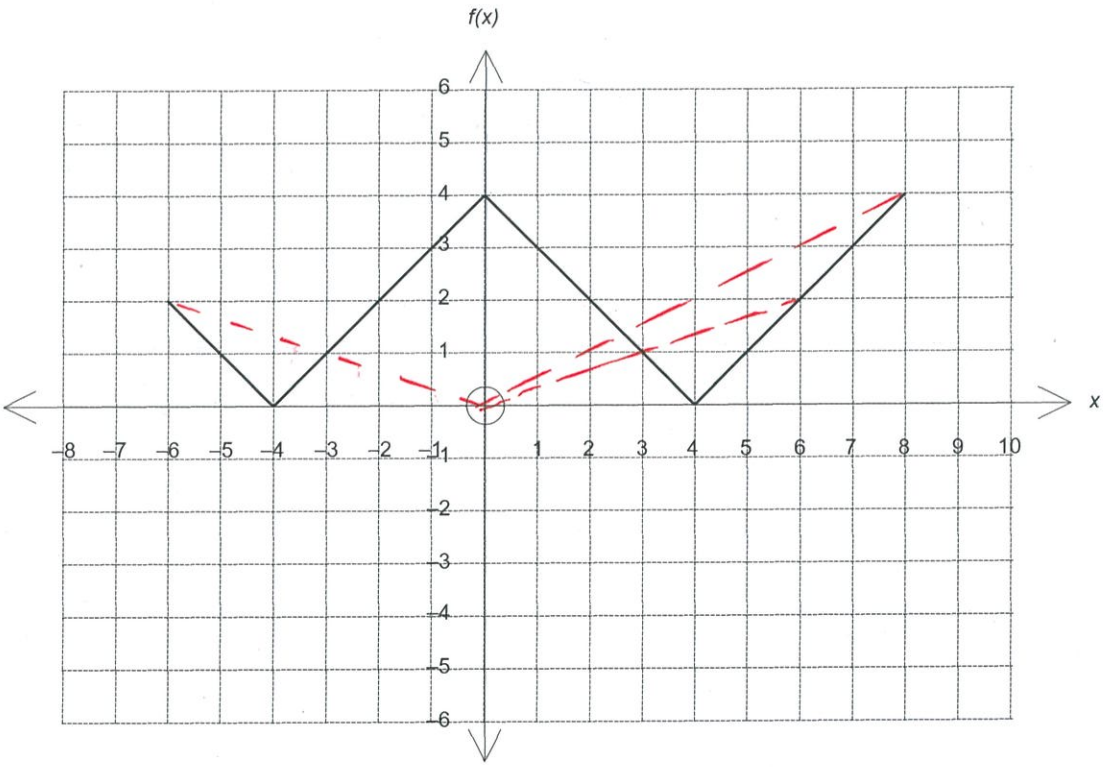
$= \frac{27}{22}\underset{\sim}{i} + \frac{39}{22}\underset{\sim}{j} + \frac{13}{11}\underset{\sim}{k}$ ✓ calculates P.O.I.

or $(1.23\underset{\sim}{i} + 1.77\underset{\sim}{j} + 1.18\underset{\sim}{k})$.

See next page

Question 6 **(8 marks)**

The grid below shows the graph of $f(x)$ for the domain $-6 \leq x \leq 8$.



(a) State the equation of $f(x)$. (2 marks)

$f(x) = ||x| - 4|$ ✓ ✓

or $f(x) = |4 - |x||$

determines $|x|$

determines horizontal translation and $|\ |$.

(b) The function $g(x) = -|x + a| + b$ intersects $f(x)$ for $-4 \leq x \leq -2$.
Determine the value of the constants a and b. (2 marks)

$g(x) = -|x+2| + 2$

$\therefore\ a = 2,\ b = 2$

correctly calculates 'a' and 'b' must be explicitly stated, otherwise award 1 mark.

(c) State the conditions on the positive constants m and n so that the graph of $h(x) = m|x| + n$ intersects the graph of $f(x)$ exactly four times. (4 marks)

For 4 P.O.I → no horizontal translation ∴ 'm' must be wide dilation. So $0 < m < k$ and must hit graph twice when $x > 0$, see - - - but when $x < 0$ stops at $(-6, 2)$ so gradient $= \frac{2}{6} = \frac{1}{3}$.

$0 < m < \frac{1}{3}$ ✓ ✓

one for each limit and correct inequality.

If $m = \frac{1}{3}$ then $n = 0$

If $m = 0$ then $n = 2$

$\therefore\ 0 < n < 2$ ✓ ✓

Do not penalise if do not include $0 <$, $0 <$

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Christ Church Grammar School

2020
TEST 2

MATHEMATICS SPECIALIST Year 12

Section One:
Calculator-free

Your name SOLUTIONS

Teacher's name _____

Time and marks available for this section

Reading time for this section:	2 minutes
Working time for this section:	25 minutes
Marks available:	27 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet using a blue/black pen. Do not use erasable or gel pens

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

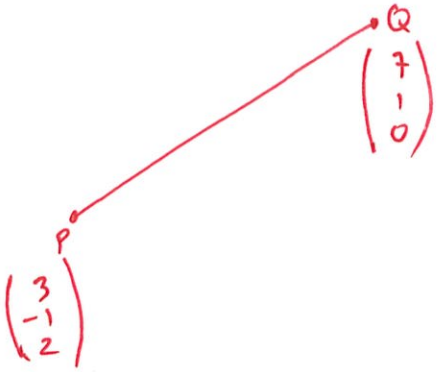
7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(6 marks)**

Points P and Q have respective position vectors $\begin{pmatrix}3\\-1\\2\end{pmatrix}$ and $\begin{pmatrix}7\\1\\0\end{pmatrix}$.

(a) Determine the vector equation of the sphere that has PQ as diameter. (3 marks)



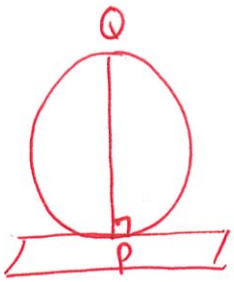
centre is midpoint $= \begin{pmatrix}5\\0\\1\end{pmatrix}$ ✓ correctly calculates midpoint

radius $= \sqrt{2^2+1^2+1^2}$

$= \sqrt{6}$ ✓ calculates radius correctly based on their midpoint

$\left| \underset{\sim}{r} - \begin{pmatrix}5\\0\\1\end{pmatrix} \right| = \sqrt{6}$ ✓ states equation based on their information.

(b) Determine the Cartesian equation of the plane that is tangent to the sphere in part (a) at the point P. (3 marks)



$\overrightarrow{PQ}$ is ⊥ to plane $\therefore \underset{\sim}{n} = \begin{pmatrix}4\\2\\-2\end{pmatrix}$ or $\underset{\sim}{n} = \begin{pmatrix}2\\1\\-1\end{pmatrix}$

✓ correctly calculates normal vector

$\underset{\sim}{r}.\underset{\sim}{n} = \underset{\sim}{a}.\underset{\sim}{n}$

$\underset{\sim}{r}.\begin{pmatrix}4\\2\\-2\end{pmatrix} = \begin{pmatrix}3\\-1\\2\end{pmatrix}.\begin{pmatrix}4\\2\\-2\end{pmatrix}$ or $\underset{\sim}{r}\begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}3\\-1\\2\end{pmatrix}.\begin{pmatrix}2\\1\\-1\end{pmatrix}$

$\underset{\sim}{r}.\begin{pmatrix}4\\2\\-2\end{pmatrix} = 12-2-4$ $\underset{\sim}{r}.\begin{pmatrix}2\\1\\-1\end{pmatrix} = 3$

$\underset{\sim}{r}.\begin{pmatrix}4\\2\\-2\end{pmatrix} = 6$ Determines correct vector equation of plane ✓

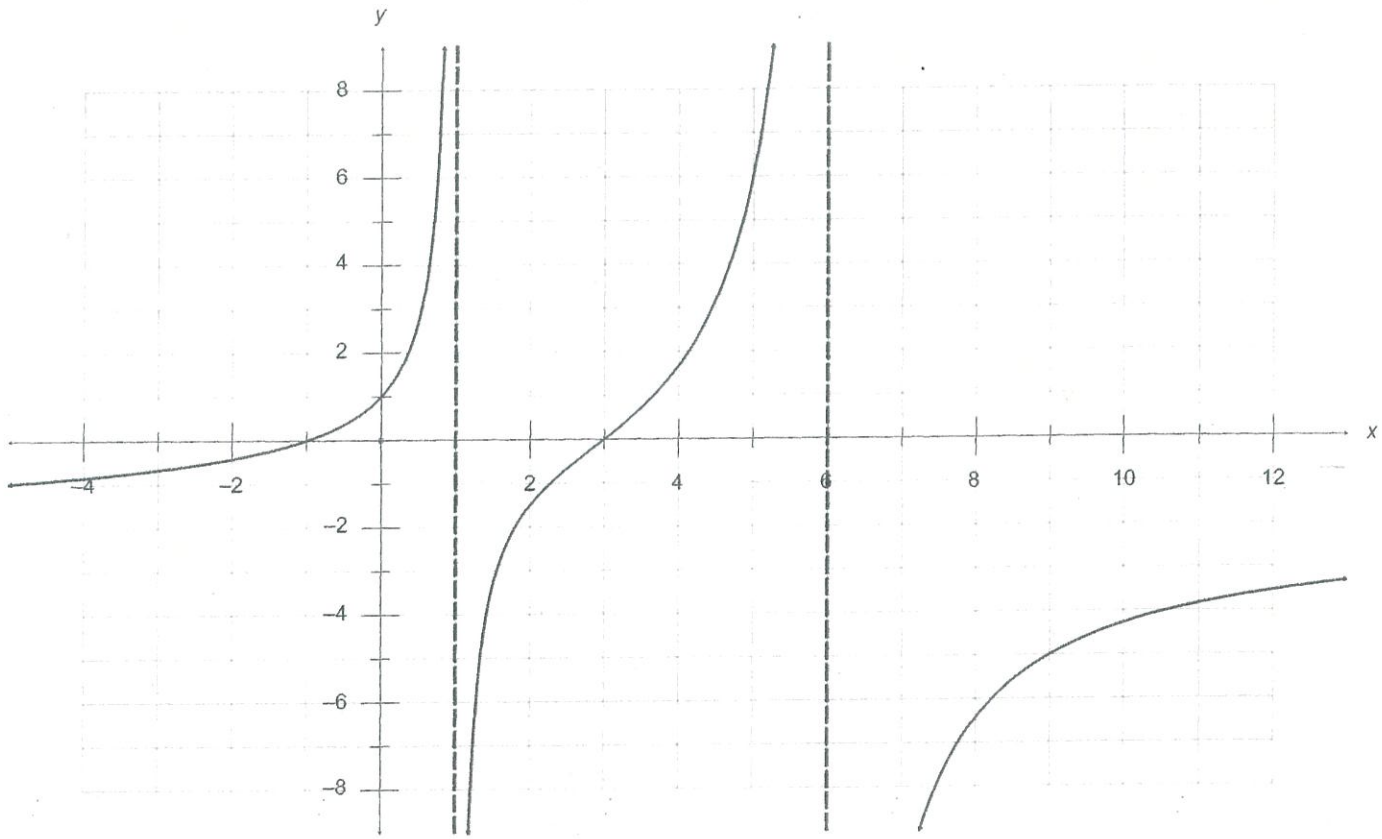
$\Rightarrow 4x+2y-2z=6$ $2x+y-z=3$

✓ converts to Cartesian equation

See next page

Question 2 **(11 marks)**

The function $f(x) = \frac{k(x+a)(x+b)}{(x+c)(x+d)}$ is shown below, where $a, b, c, d, k \in \mathbb{R}$



(a) State the value of the constants a, b, c, d and k. (5 marks)

VA : $x = 1$, $x = 6$ $\therefore$ $c = -1$ $d = -6$
or $c = -6$ $d = -1$
✓✓ correctly calculates c, d.

Roots : $x = 3$, $x = -1$ $\therefore$ $a = 1$, $b = -3$
or $a = -3$, $b = 1$
✓✓ correctly calculates a, b

y-int at 1. $1 = \frac{k(1)(-3)}{(-1)(-6)}$

$1 = \frac{-3}{6}k$.

$\therefore$ $k = -2$ ✓ calculates based on their a, b, c, d

So $a = 1$, $b = -3$, $c = -1$, $d = -6$, $k = -2$ or correct combinations

See next page

Question 2 continued

(b) Draw the graph of $\frac{1}{f(x)}$ on the grid provided below. (5 marks)

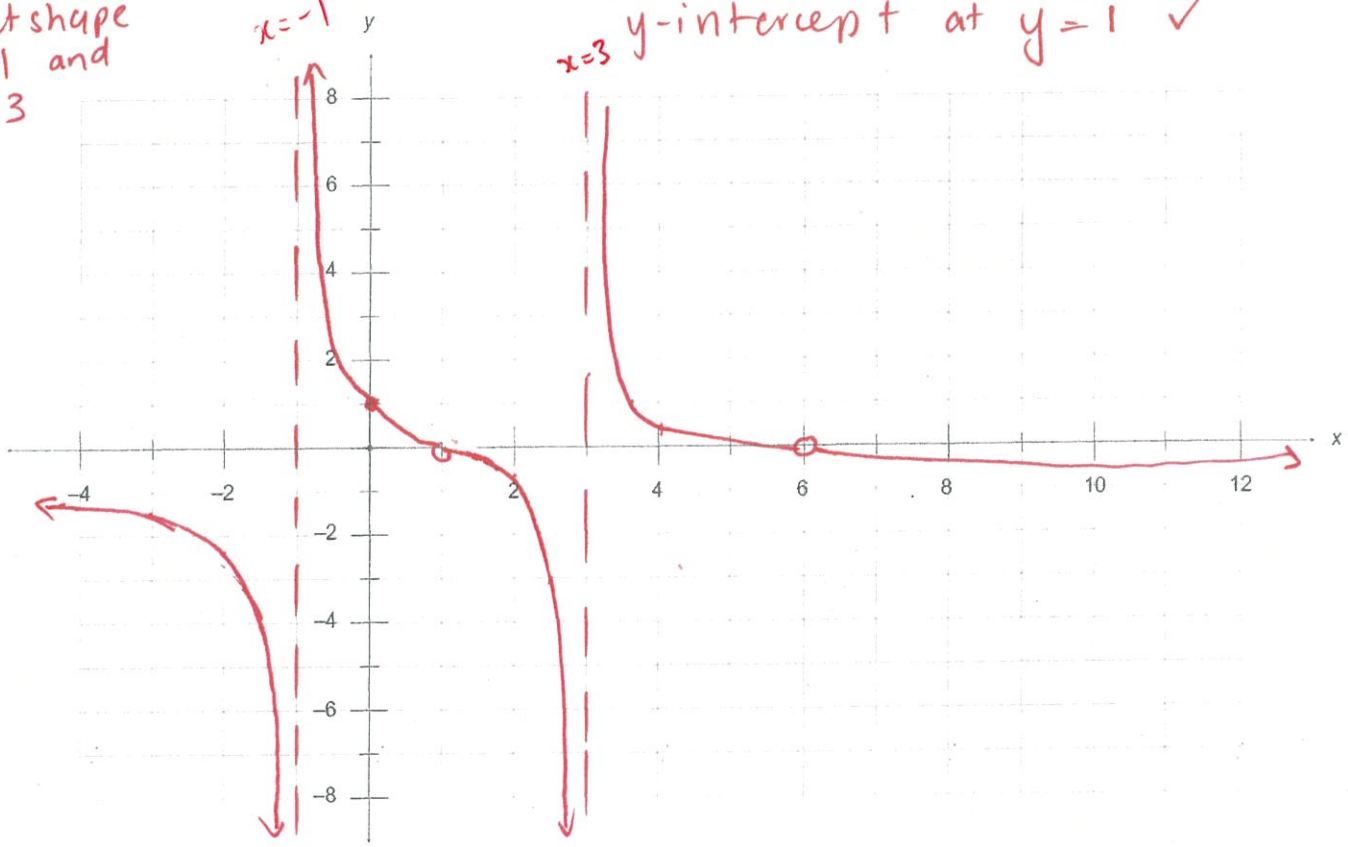
✓ correct shape b/n VA.

✓ correct shape $x<-1$ and $x>3$

VA : $x=-1$, $x=3$ ✓

discontinuities at $x=1, 6$ ✓

y-intercept at $y=1$ ✓



move down.

(c) Solve the equation $f(|x|) = 0$. (1 mark)

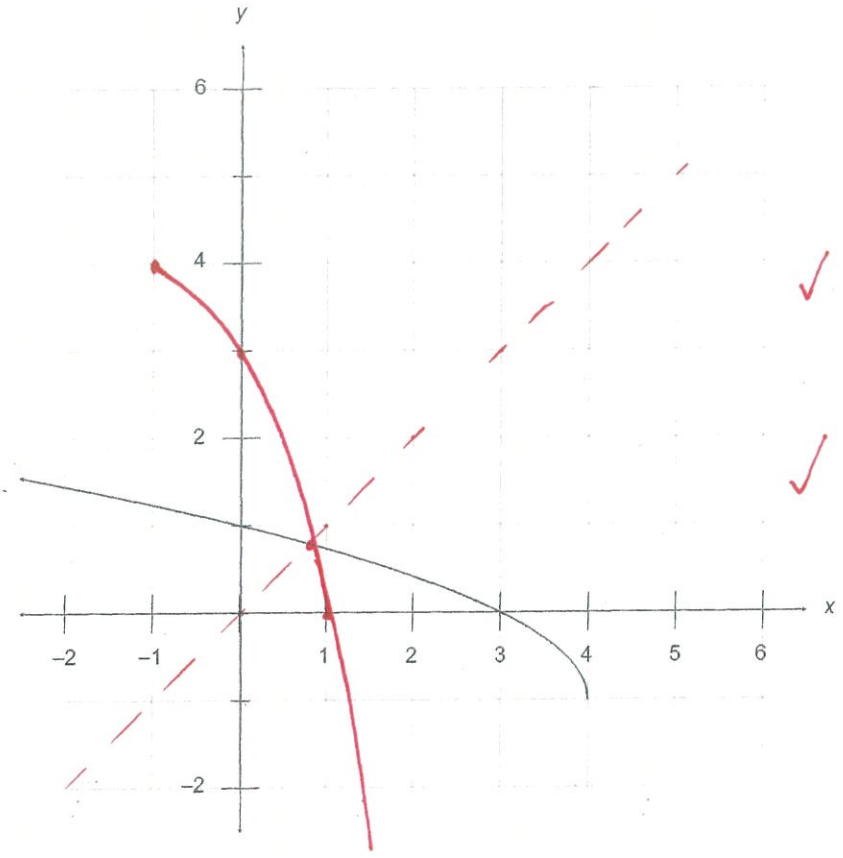
$x = \pm 3$ ✓

Note : $f(x) = 0$ when $x > 0$ at $x = 3$

$\therefore$ repeats graph when $x < 0$. so also at $x = -3$.

Question 3 **(10 marks)**

The function $f(x)$ is defined at $f(x) = \sqrt{4-x} - 1$ and its graph is shown below.



✓ intersects $f(x)$ along the line $y = x$

✓ correct shape and location of intercepts

(a) Sketch the graph of $f^{-1}(x)$ on the same grid above. (2 marks)

(b) Determine the equation of $f^{-1}(x)$ and state its domain. (3 marks)

$y = \sqrt{4-x} - 1$ Range: $y \geq -1$

$y + 1 = \sqrt{4-x}$

$(y+1)^2 = 4 - x$

$x = 4 - (y+1)^2$

✓ 2 correct steps in re-arrangement.

$\therefore f^{-1}(x) = 4 - (x+1)^2$

or $f^{-1}(x) = -x^2 - 2x + 3$

✓ correct inverse function

Domain: $\{x \in \mathbb{R} : x \geq -1\}$

✓ correct domain (same as $f(x)$ range)

See next page

Question 3 continued

The function $g(x)$ is defined as $g(x) = 4 - x^2$.

(c) Determine an expression for $f \circ g(x)$. (1 mark)

$fog(x) = \sqrt{4-(4-x^2)} - 1$

$= \sqrt{x^2} - 1$

$= |x| - 1$ ✓

correct answer
accept either

(d) Does the domain of $g(x)$ need to be adjusted for $f \circ g(x)$ to exist? Justify your answer and determine the corresponding range of the composition. (4 marks)

For $fog(x)$ to exist $\sqrt{\quad} \geq 0$

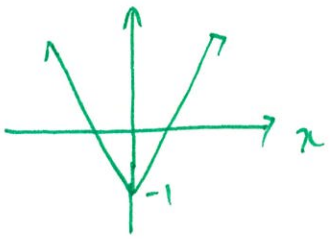
$\therefore 4 - g(x) \geq 0$

$4 - (4 - x^2) \geq 0$

$x^2 \geq 0$

$\therefore x \in \mathbb{R}$ ✓ determines correct domain for x.

statement that $\sqrt{\quad} \geq 0$
accept any of these
or if graphically



So no changes to $g(x)$ domain required ✓ makes correct statement

$R: \{ y \in \mathbb{R} : y \geq -1 \}$ ✓

determines correct range.

Note: if uses graph to find range then must show graph (with labels) to get marks.

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____